

Non-Minimal Inflation after WMAP3

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Abstract

The Wilkinson Microwave Anisotropy Probe (WMAP) three year results are used to constraint non-minimal inflation models. Two different non-minimally coupled scalar field potentials are considered to calculate corresponding slow-roll parameters of non-minimal inflation. The results of numerical analysis of parameter space are compared with WMAP3 data to find appropriate new constraints on the values of the non-minimal coupling. A detailed comparison of our results with previous studies reveals the present status of the non-minimal inflation model after WMAP3.

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1 Motivation

Non-minimal coupling (NMC) of scalar field with gravity is necessary in many situations of physical and cosmological interest. There are several compelling reasons to include an explicit non-minimal coupling in the action (see for example [1,2,3,4] and references therein). NMC arises at the quantum level when quantum corrections to the scalar field theory are considered. It is necessary also for the renormalizability of the scalar field theory in curved space. In most theories used to describe inflationary scenarios, it turns out that a non-vanishing value of the coupling constant is unavoidable [2]. In general relativity, and in all other metric theories of gravity (in which the scalar field is not part of the gravitational sector), the coupling constant necessarily assumes a non-vanishing value[5]. The study of the asymptotically free theories in an external gravitational field with a Gauss-Bonnet term shows a scale dependent coupling parameter. For instance, asymptotically free grand unified theories have a non-minimal coupling depending on a renormalization group parameter that converges to the value of $\frac{1}{6}$ or to any other initial conditions depending on the gauge group and on the matter content of the theory[6]. In view of the above results and several other motivations(see for example [7,8] and references therein), it is then natural to incorporate an explicit NMC between scalar field and Ricci scalar in the inflationary paradigm. Generally, with non-minimally coupled scalar field it is harder to achieve accelerated expansion of the universe[2,7]. Part of this difficulty is related to the more sophisticated machinery of fine tuning. Nevertheless, over the last decade several non-minimal inflation scenario have been proposed to find reliable framework for issues such as graceful exit of inflationary phase and the observational constraints on the values that non-minimal coupling can attain in order to have successful inflationary scenario are discussed [8-16].

The recent astronomical observations, specially high precision data of WMAP3 [17] have opened new doors in the field of observational cosmology. As a result, these data have significant impact on the inflation paradigm. In this regard, these high precision data will give more accurate bounds on the values of non-minimal coupling in a typical non-minimal inflation model. The purpose of this letter is to study impact of WMAP3 and non-minimal inflation. Considering some well-known inflationary potentials, we explore new observational constraints on the values of non-minimal coupling to have successful non-minimal inflation. By definition of an effective scalar field potential, we show that there is a region in parameter space that inflation is driven by the non-minimal coupling term. A detailed study of spectral index and its running shows the spontaneous exit of inflationary phase (without any mechanism) in a suitable region of the parameter space.

We also compare our results with the results of previous studies. This comparison reveals the present status of non-minimal inflation paradigm after WMAP3.

2 Non-Minimal Inflation

To construct a specific non-minimal inflation scenario, we start with the following action in Jordan frame

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} \xi R \phi^2 + V(\phi) \right] \quad (1)$$

where $\frac{\kappa^2}{8\pi} \equiv G = m_{\text{pl}}^{-2}$ is Newton's gravitational constant, and ξ is a coupling constant. The metric signature convention is chosen to be $(+ - - -)$ with spatially flat Robertson-Walker metric as follows

$$ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j. \quad (2)$$

To obtain the fundamental background equations in Einstein frame, we perform the following conformal transformation to the Einstein frame

$$\hat{g}_{\mu\nu} = \Omega g_{\mu\nu}, \quad \Omega = 1 + \kappa^2 \xi \phi^2. \quad (3)$$

Here we use a hat on a variable defined in the Einstein frame. The conformal transformation gives

$$S = \int d^4x \sqrt{-\hat{g}} \left[\frac{1}{2\kappa^2} \hat{R} - \frac{1}{2} F^2(\phi) \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \hat{V}(\phi) \right], \quad (4)$$

where by definition

$$F^2(\phi) \equiv \frac{1 + \kappa^2 \xi \phi^2 (1 + 6\xi)}{(1 + \kappa^2 \xi \phi^2)^2} \quad (5)$$

and

$$\hat{V}(\phi) \equiv \frac{V(\phi)}{(1 + \kappa^2 \xi \phi^2)^2}. \quad (6)$$

Therefore, one may redefine the scalar field as follows

$$\frac{d\hat{\phi}}{d\phi} = F(\phi) = \frac{\sqrt{1 + \kappa^2 \xi \phi^2 (1 + 6\xi)}}{1 + \kappa^2 \xi \phi^2}. \quad (7)$$

When we investigate the dynamics of universe in the Einstein frame, we should transform our coordinates system to make the metric in the Robertson-Walker form

$$\hat{a} = \sqrt{\Omega} a, \quad d\hat{t} = \sqrt{\Omega} dt, \quad (8)$$

and we obtain

$$d\hat{s}^2 = d\hat{t}^2 - \hat{a}^2(\hat{t})\delta_{ij}d\hat{x}^i d\hat{x}^j. \quad (9)$$

Note that the physical quantities in Einstein frame should be defined in this coordinate system. Now the Einstein equations can be written as follows

$$\hat{H}^2 = \frac{\kappa^2}{3} \left[\left(\frac{d\hat{\phi}}{d\hat{t}} \right)^2 + \hat{V}(\hat{\phi}) \right], \quad (10)$$

$$\frac{d^2\hat{\phi}}{d\hat{t}^2} + 3\hat{H}\frac{d\hat{\phi}}{d\hat{t}} + \frac{d\hat{V}}{d\hat{\phi}} = 0. \quad (11)$$

Using the slow-roll approximations $\dot{\hat{\phi}}^2 \ll \hat{V}$ and $\ddot{\hat{\phi}} \ll 3\hat{H}\dot{\hat{\phi}}$, we obtain from (10) and (11) the following dynamical equations

$$\hat{H}^2 = \frac{\kappa^2}{3} \hat{V}(\hat{\phi}), \quad (12)$$

$$3\hat{H}\frac{d\hat{\phi}}{d\hat{t}} + \frac{d\hat{V}}{d\hat{\phi}} = 0. \quad (13)$$

In which follows we will concentrate on two well-known inflationary potentials: models with inflation potential of the form $V(\phi) = \lambda\phi^n$ and models with an exponential potential $V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{pm_{pl}^2}}\phi\right)$. With these potentials, we obtain

$$\hat{H}^2 \approx \frac{\kappa^2}{3} \frac{\lambda\phi^n}{(1 + \kappa^2\xi\phi^2)^2}, \quad (14)$$

$$\hat{H}^2 \approx \frac{\kappa^2}{3} \frac{V_0 \exp\left(-\sqrt{\frac{16\pi}{pm_{pl}^2}}\phi\right)}{(1 + \kappa^2\xi\phi^2)^2}. \quad (15)$$

respectively. Figure 1 shows the behavior of effective potentials as defined by relation (6). For $V(\phi) = \lambda\phi^n$ it is possible to achieve inflationary phase with appropriate ξ . For $V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{pm_{pl}^2}}\phi\right)$ however, when ξ is negative, since the effective potential is flat in region of $\phi > 0$ compared with the $\xi = 0$ case, we can expect assisted inflation to occur in this region. When ξ is positive, inflation can be realized in the region of $\phi < 0$. For a detailed study of these issues for power-law inflation see [18].

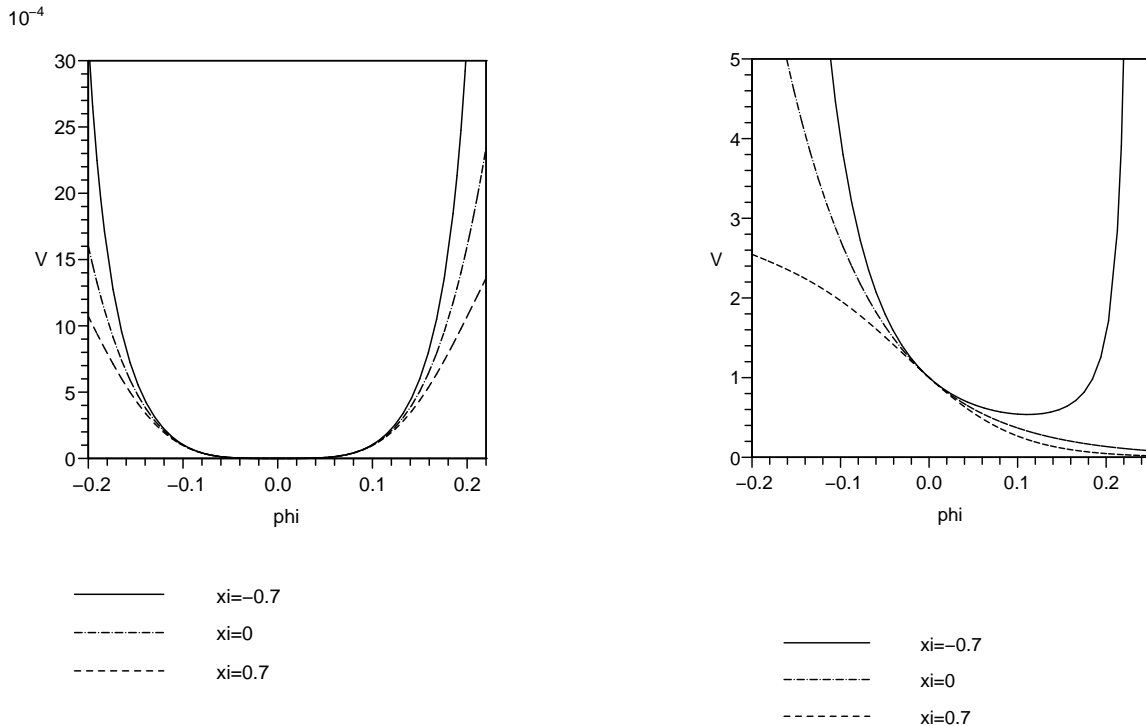


Figure 1: Variation of \hat{V} relative to ϕ for different values of ξ for potential $\lambda\phi^n$ with $n = 4$ (left) and exponential potential (right).

Note that we have used a well-known conformal transformation to express the action in the Einstein frame. In this frame, the gravitational sector is expressed in terms of a re-scaled scalar field which is minimally coupled and evolves in a re-scaled potential, thereby simplifying the formalism. However, one has to keep in mind that the matter sector is strongly affected by such a conformal transformation since all of the matter fields are now non-minimally coupled to the re-scaled metric: in particular, stress tensor conservation in the matter sector is no longer ensured [19,20]. On the other hand, as Makino and Sasaki [21] and Fakir, Habib and Unruh [22] have proved, the amplitude of scalar perturbation in the Jordan frame exactly coincides with that in the Einstein frame. This proof (with a complete details in [23]) allows us to calculate the scalar power spectrum in the Jordan and Einstein frame. As a result, the scalar power spectrum has not dependence on the choice of frames, that is, it is conformally invariant. So, our results can be compared to observations directly without any ambiguities[23]. This is an important point since one has to check validity of non-gravity experiments in Einstein frame. For instance, validity of electro-magnetic related experiments such as CMB experiment should be checked in Einstein frame. As Komatsu and Futamase have shown, the scalar power spectrum is independent on the choice of frames.

Now we study the effect of non-minimal coupling of scalar field and curvature on the spectral index of density perturbations.

2.1 Slow-Roll Parameters

In this section we calculate the spectral index of density perturbations in our non-minimal framework. As Komatsu and Futamase have shown, in the first order approximation in slow-roll parameters, n_s is invariant under the conformal transformation [23]. We use the following definitions of slow-roll parameters versus $\hat{\phi}$ as defined by Liddle *et al*[24] (see also [25-27]):

$$\epsilon \equiv \frac{1}{2\kappa^2} \left(\frac{\hat{V}'(\hat{\phi})}{\hat{V}(\hat{\phi})} \right)^2, \quad \eta \equiv \frac{1}{\kappa^2} \left(\frac{\hat{V}''(\hat{\phi})}{\hat{V}(\hat{\phi})} \right), \quad (16)$$

and

$$\zeta \equiv \frac{1}{\kappa^2} \left(\frac{\hat{V}'(\hat{\phi})\hat{V}'''(\hat{\phi})}{\hat{V}^2(\hat{\phi})} \right)^{\frac{1}{2}}. \quad (17)$$

where a prime denotes $\frac{d}{d\hat{\phi}}$. These parameters can be related directly to observable $n_s - 1 = -6\epsilon + 2\eta$ [24-27]. Of course, to the second order of slow-roll parameters, the spectral index will attain different form[25]. Now we use equation (16), (6) and (7) to determine n_s . Since, $\hat{V}' = \frac{d\hat{V}}{d\hat{\phi}} = \frac{d\phi}{d\hat{\phi}} \frac{d\hat{V}}{d\phi}$, we find

$$\epsilon = \frac{1}{2\kappa^2\phi^2} \frac{[n + \kappa^2\xi\phi^2(n-4)]^2}{[1 + \kappa^2\xi\phi^2(1+6\xi)]}, \quad (18)$$

$$\begin{aligned} \eta = & \frac{1}{\kappa^2} \frac{1}{(1 + \kappa^2\xi\phi^2)^2} \left[(1 + \kappa^2\xi\phi^2)(n(n+1)\phi^{-2} + n(n+3)\kappa^4\xi^2\phi^2 + 2n(n+1)\kappa^2\xi - 4(n+1)\kappa^2\xi \right. \\ & \left. - 4(n+3)\kappa^4\xi^2\phi^2 - 8\kappa^2\xi(n + n\kappa^4\xi^2\phi^4 + 2n\kappa^2\xi\phi^2 - 4\kappa^2\xi\phi - 4\phi^3\kappa^4\xi^2)) \left(\frac{1}{(1 + \kappa^2\xi\phi^2(1+6\xi))^{\frac{1}{2}}} \right) \right] \\ & + \frac{1}{\kappa^2} \times \frac{2\kappa^2\xi\phi(1 + \kappa^2\xi\phi^2(1+6\xi))^{\frac{1}{2}} - (1 + 2\kappa^2\xi\phi^2)\kappa^2\xi\phi(1+6\xi)(1 + \kappa^2\xi\phi^2(1+6\xi))^{-\frac{1}{2}}}{(1 + \kappa^2\xi\phi^2(1+6\xi))} \\ & \times \frac{(n\phi^{-1}(1 + \kappa^2\xi\phi^2)^2 - 4\kappa^2\xi\phi(1 + \kappa^2\xi\phi^2))}{(1 + \kappa^2\xi\phi^2)^2}. \end{aligned} \quad (19)$$

for $V(\phi) = \lambda\phi^n$. For potential $V(\phi) = V_0 \exp(-\sqrt{\frac{16\pi}{pm_{pl}^2}}\phi)$ we derive

$$\epsilon = \frac{1}{2\kappa^2} \left(\frac{-4\xi\kappa^2\phi - \sqrt{\frac{16\pi}{pm_{pl}^2}}(1 + \xi\kappa^2\phi^2)}{\sqrt{1 + (1 + 6\xi)\xi\kappa^2\phi^2}} \right)^2, \quad (20)$$

$$\eta = \frac{1}{2\kappa^2} \left(\frac{(-4\xi\kappa^2 + 4\xi\kappa^2\phi\sqrt{\frac{16\pi}{pm_{pl}^2}}) + (-2\xi\kappa^2\phi\sqrt{\frac{16\pi}{pm_{pl}^2}} + \frac{16\pi}{pm_{pl}^2}(1 + \xi\kappa^2\phi^2))}{(1 + \xi\kappa^2\phi^2)^2(1 + \xi\kappa^2\phi^2(1 + 6\xi))} - \left[\frac{(-4\xi\kappa^2\phi - \sqrt{\frac{16\pi}{pm_{pl}^2}}(1 + \xi\kappa^2\phi^2))(4\xi\kappa^2\phi(1 + \xi\kappa^2\phi^2)(1 + \xi\kappa^2\phi^2(1 + 6\xi))}{(1 + \xi\kappa^2\phi^2)^2(1 + \xi\kappa^2\phi^2(1 + 6\xi))} + \frac{(1 + 6\xi)\xi\kappa^2\phi(1 + (1 + 6\xi)\xi\kappa^2\phi^2)^{-\frac{1}{2}}(1 + \xi\kappa^2\phi^2)^2}{(1 + \xi\kappa^2\phi^2)^2(1 + \xi\kappa^2\phi^2(1 + 6\xi))} \right] \right). \quad (21)$$

The inflationary phase terminates if the condition $\epsilon = 1$ be satisfied[24]. For exponential inflationary potential as described above, slow-roll equations cannot be solved in algebraic way, so we try to solve them numerically. In the case of the large field models $\lambda\phi^n$, these equations can be solved analytically, however the result of numerical calculation can be interpreted more easily. Figures 2 shows the results of these calculations for potential $V(\phi) = \lambda\phi^n$ for both positive and negative scalar field ϕ . We see that with positive ξ , both positive and negative ϕ can lead to spontaneous exit of inflationary phase without any additional mechanism. For negative ξ however, as figure 3 shows, the situation is very different. With $V(\phi) = \lambda\phi^4$ and negative ϕ as figure 3 (left) shows, inflationary phase can exit only for $\xi \leq -0.1666$. For $\xi \leq -0.1666$ inflationary phase can exit spontaneously without any mechanism. For all $\xi > -0.1666$ we find a negative ϵ which is evidently impossible since ϵ is a positive definite quantity. On the other hand, with positive ϕ and negative ξ as figure 3 (right) shows, inflationary phase can exit spontaneously only for $\xi \leq -\frac{1}{6}$. For $V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{pm_{pl}^2}}\phi\right)$, figures 4 and 5 show that inflationary phase can exit spontaneously with all possible values of ξ regardless of ϕ sign. So, non-minimal coupling provides a successful machinery for spontaneous exit of inflationary phase.

The number of e-folds, $N \equiv \ln \frac{a_e}{a_i}$ can be written as

$$N = - \int_{\hat{\phi}_i}^{\hat{\phi}_e} 3\hat{H}^2 \frac{d\hat{\phi}}{d\hat{V}} d\hat{\phi} = - \sqrt{\frac{4\pi}{m_{pl}^2}} \left| \int_{\hat{\phi}_i}^{\hat{\phi}_e} \epsilon^{-\frac{1}{2}} d\hat{\phi} \right|. \quad (22)$$

where $\hat{\phi}_i$ denotes the value of scalar field $\hat{\phi}$ when Universe scale observed today crosses the Hubble horizon during inflation, while $\hat{\phi}_e$ is the value of scalar field when the Universe exits the inflationary phase. The number of e-folds can be obtained using equations (6) and (10). Now we set $n = 4$ and for simplicity we choose $\lambda = \frac{\lambda'}{4}$ in $V(\phi) = \lambda\phi^n$. Writing $\kappa^2\xi\phi_{end}^2 = \beta^2(\xi)$ in relation (18), we can solve equation $\epsilon = 1$ for $\beta(\xi)$ to find

$$\beta = \sqrt{\frac{1}{2(1 + 6\xi)}} (\sqrt{192\xi^2 + 32\xi + 1} - 1). \quad (23)$$

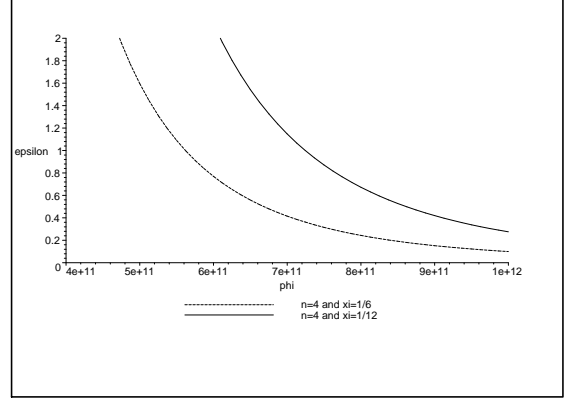
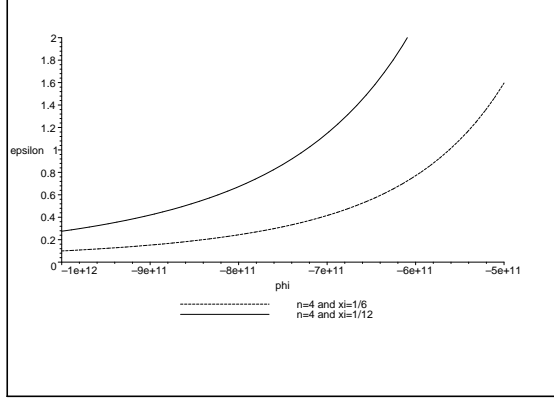
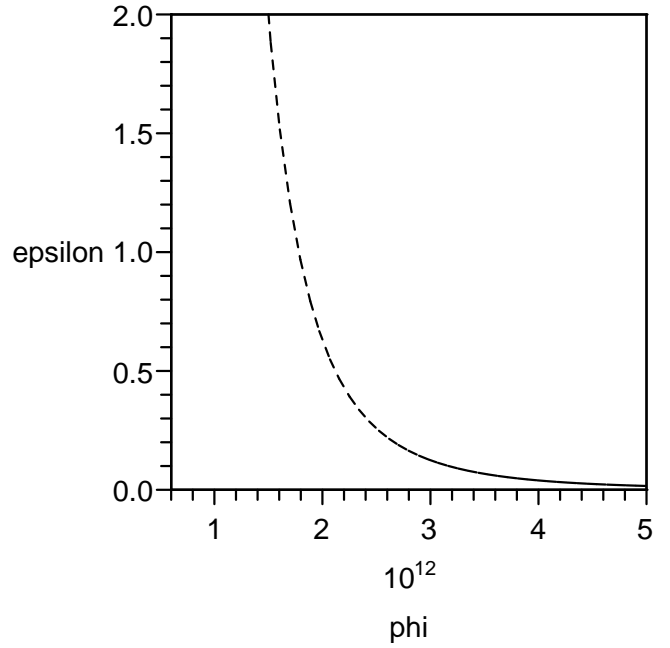
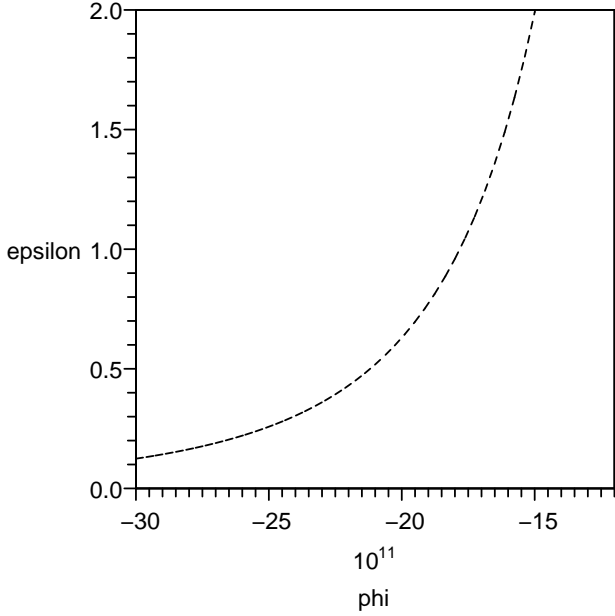


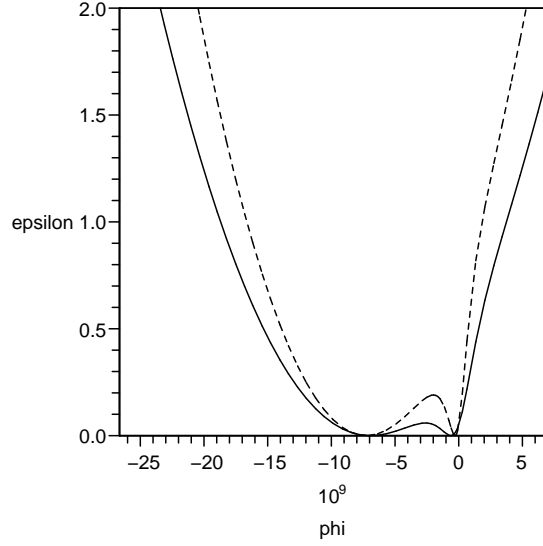
Figure 2: Variation of ϵ for different values of (positive) non-minimal coupling for potential $\lambda\phi^n$ with $n = 4$. In both cases, it is possible to exit inflationary phase without any additional mechanism.



----- $n=4$ and $\xi=-1/6$

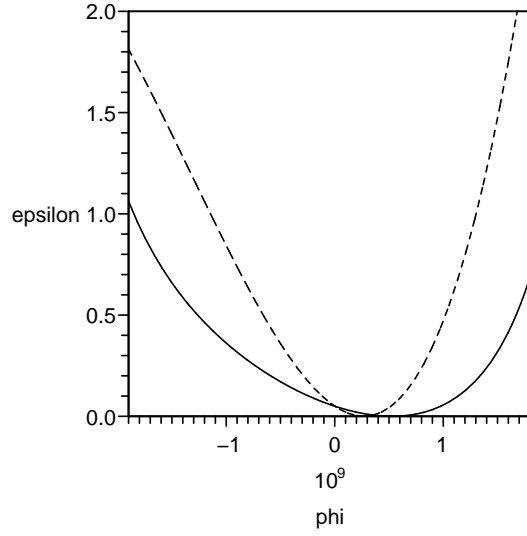
----- $n=4$ and $\xi=-1/6$

Figure 3: Variation of ϵ for different values of (negative) non-minimal coupling for potential $\lambda\phi^n$ with $n = 4$. In both cases, it is possible to exit inflationary phase spontaneously only with suitable range of non-minimal coupling.



----- p=20 and xi=1/6
 ————— p=20 and xi=1/12

Figure 4: Variation of ϵ for different values of (positive) non-minimal coupling for potential $V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{pm_{pl}^2}}\phi\right)$. In both cases, it is possible to exit inflationary phase without any additional mechanism.



----- p=20 and xi=-1/6
 ————— p=20 and xi=-1/12

Figure 5: Variation of ϵ for different values of (negative) non-minimal coupling for potential $V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{pm_{pl}^2}}\phi\right)$. In both cases, it is possible to exit inflationary phase without any additional mechanism.

If we similarly write $\kappa^2 \xi \phi_{hc}^2 = m^2(\xi)$ where ϕ_{hc} is the value of ϕ when the scales of interest crossed outside of horizon during inflation, then we can rewrite relations (18) and (19) using the fact that scales of interest to us crossed outside of horizon is approximately 70 e-folds before the end of inflation, that is, $e^N \equiv \frac{\hat{a}(t_{end})}{\hat{a}(t_{hc})} \sim e^{70}$. The first-order result, $n_s = 1 - 6\epsilon + 2\eta$, can be written approximately, in the limit of $m \gg 1$, as follows

$$n_s \simeq 1 - \frac{32\xi}{16 \times 70\xi - 1}. \quad (24)$$

The running of the spectral index is therefore

$$\alpha_s = \frac{dn_s}{d \ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\zeta \simeq -2\zeta \approx -10^{-2} \quad (25)$$

where $k = \hat{a}\hat{H}$. This quantity could be larger and match observation if we adopt a larger value of the coupling constant λ (for example $\alpha_s = -0.021$ if $\lambda = 0.30$). Figure 6 shows the result of this calculation. As we see, both of these figures show that $n_s \leq 1$ and therefore our model favors a red and nearly scale invariant spectrum.

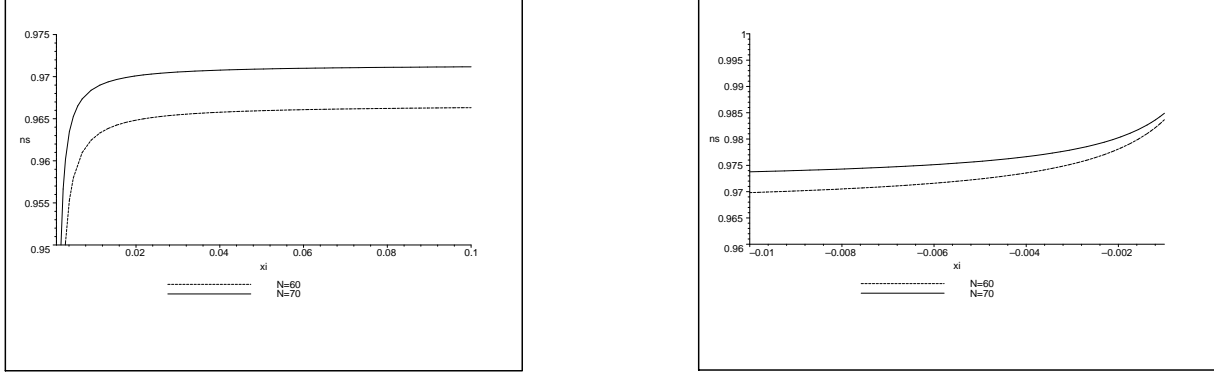


Figure 6: Variation of spectral index, n_s for different values of non-minimal coupling for potential $\lambda\phi^n$. Evidently, $n_s \leq 1$ and therefore our non-minimal model favors a red power spectrum.

We can similarly calculate n_s and the running of the spectral index for exponential potential. The results are as follows

$$n_s = 1 - \frac{3 \left(-4a\sqrt{\xi} - (1+a)\sqrt{\frac{2}{p}} \right)^2}{1 + (1+6\xi)a}, \quad (26)$$

where $p \gg 2$ and $a \sim -\frac{m_{pl}}{\sqrt{8\pi\xi}}$. The running of the spectral index is

$$\alpha_s = \frac{dn_s}{d \ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\zeta \simeq -2\zeta \approx -10^{-1}, \quad (27)$$

for example $\alpha_s = -0.102$ if $p = 14$. In this case we cannot consider $\xi < 0$, since with $\xi < 0$ relation (26) for spectral index becomes a complex quantity. Figure (7) shows the variation of spectral index for exponential potential. We see that in this case we have always $n_s \leq 1$. Therefore, our model for exponential potential gives also a nearly scale invariant spectrum.

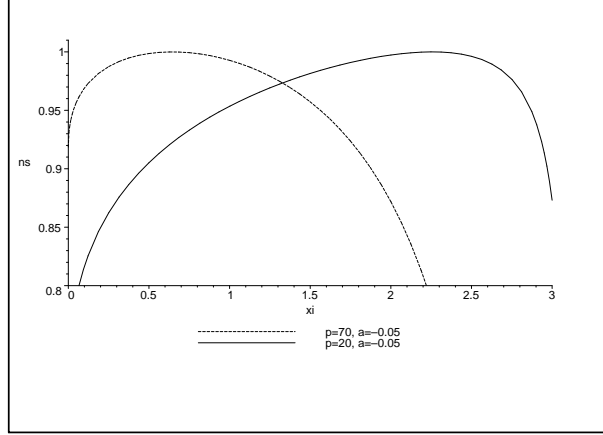


Figure 7: Variation of n_s for different values of non-minimal coupling for potential $V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{p m_{pl}^2}}\phi\right)$.

3 Numerics of Parameter Space and Observational Constraints

To study cosmological implications of this non-minimal inflation model, we perform some numerical analysis of parameter space. The result of WMAP3 for Λ CDM gives $n_s = 0.951^{+0.015}_{-0.019}$ for index of the power spectrum[17]. Combining WMAP3 with SDSS (Sloan Digital Sky Survey), gives $n_s = 0.948^{+0.015}_{-0.018}$ at the level of one standard deviation[12]. These results show that a red power spectrum is favored at least at the level of two standard deviations. If there is running of the spectral index, the constraints on the spectral index and its running are given by [12]

$$n_s = 1.21^{+0.13}_{-0.16} \quad (28)$$

and

$$\frac{dn_s}{d \ln k} = -0.102^{+0.050}_{-0.043}. \quad (29)$$

Table 1 summarizes the results of our calculations for spectral index for two different kind of scalar field potential. This table contains also the constraints imposed on the non-minimal coupling in comparison with WMAP3 data. Choosing some limiting values of α_s , we obtain a suitable range of non-minimal coupling. In this table, $a \sim -\frac{m_{pl}}{\sqrt{8\pi\xi}}$ is the value of the scalar field when the inflationary phase terminates. Therefore, to have assisted inflation with non-minimal coupling, we should restrict ourselves to the conditions $\xi \leq -0.1666$ and $\xi \geq 0.01$ for $V(\phi) = \lambda\phi^n$ and $0.271 \leq \xi \leq 0.791$ for exponential potential. Since the values that n_s can attain in our non-minimal framework are less than unity, our model shows a red power spectrum in accordance with WMAP3. On the other hand, our model shows a nearly scale invariant spectrum.

Table 1: Constraining Non-minimal inflation with WMAP3

$V(\phi)$	n_s	α_s	ξ
$\frac{\lambda}{4}\phi^4$	$1 - \frac{32\xi}{16 \times 70\xi - 1}$	$\sim -10^{-2}$	$\xi \leq -0.1666$ and $\xi \geq 0.01$
$V_0 \exp(-\sqrt{\frac{16\pi}{pm_{pl}^2}}\phi)$	$n_s = 1 - \frac{3(-4a\sqrt{\xi} - (1+a)\sqrt{\frac{2}{p}})^2}{1 + (1+6\xi)a}$	$\sim -10^{-1}$	$0.271 \leq \xi \leq 0.791$

At this stage we compare our results with the previous studies. Table 2 summarizes the results of previous studies in comparison with our present results. As we see, our constraints for non-minimal coupling with exponential potential are consistent with holographic dark energy framework and also with result of warped DGP brane inflation. On the other hand, with potential of the form $V(\phi) = \lambda\phi^n$, we find a more precise interval of the non-minimal coupling for assisted inflation. It is well-known that multiple scalar fields with exponential potential can lead to an inflationary solution even if each scalar field alone fails to provide this situation. Here we see that with non-minimally coupled scalar field the situation is different and non-minimal coupling itself can assist the inflationary phase. So, inflation can be assisted by non-minimal coupling even in the one field case. In this framework, primordial perturbations are almost scale-independent. The primordial power spectrum predicted by this non-minimal inflation is almost scale-independent, that is the spectral index n_s is very close to unity. Possible deviations from exact scale independence arise because during inflation the inflaton is not massless and the Hubble

rate is not exactly constant. As we know, the WMAP3 data favors a red power spectrum at the level of two standard deviations, which provides a stringent constraint on the inflation models. While allowing for a running spectral index slightly improves the fit to the WMAP data. With these preliminaries, our non-minimal inflation model provides a realistic framework for spontaneous exit of inflationary phase without any additional mechanism with suitable constraints imposed on the values that non-minimal coupling can attain.

In summary, with non-minimal coupling there is a region of parameter space that inflation is driven by the non-minimal coupling term alone. Also, non-minimal inflation provides a suitable mechanism for spontaneous exit of inflationary phase in some special circumstances. Our non-minimal inflation scenario gives a red and nearly scale invariant power spectrum. Comparison of non-minimal inflation model with WMAP3 data gives more accurate constraints on the values of non-minimal coupling. A detailed comparison between our results and results of previous studies shows that our constraints for non-minimal coupling with exponential potential are consistent with holographic dark energy framework and also with result of warped DGP brane inflation. On the other hand, with potential of the form $V(\phi) = \lambda\phi^n$, we find a more precise interval of the non-minimal coupling for assisted inflation. Since we have calculated the first order contributions in slow-roll parameters, our results are valid in both Jordan and Einstein frame. However, higher order corrections evidently lead to different results in these two frames[23].

Table 2: Constraints on ξ in different Non-minimal Inflation Models

<i>Model</i>	<i>Authors</i>	<i>Constraint on ξ</i>	<i>Source</i>
<i>Holographic dark energy</i> [10]	<i>M.Ito</i>	$0.146 \leq \xi \leq 0.167$	<i>HS as IR</i> ¹
<i>Quintessence Model</i> [11]	<i>T.Chiba</i>	$-10^{-2} \leq \xi \leq 10^{-2}$	<i>SSE</i> ²
<i>Scalar Field theory</i> [9]	<i>S.Koh et al</i>	$-10^{-3} < \xi < 10^{-3}$	<i>Theoretical</i>
<i>Quintessence</i> [28]	<i>S.M.Carroll</i>	$-10^{-4} \leq \xi \leq 10^{-4}$	<i>CRS</i> ³
<i>CMB Anisotropy</i> [29]	<i>T.Futamase et al</i>	$\xi > 10^{-2}$	<i>CMB</i>
<i>COBE Data Analysis</i> [30]	<i>S.Hancock et al</i>	$-10^{-4} \leq \xi \leq -10^{-3}$	<i>COBE</i>
<i>Quantum radiative Process</i> [31]	<i>A.Bilandzic et al</i>	$0.002 \ll \xi \ll 0.041$	<i>COBE</i>
<i>Density perturbations</i> [32]	<i>S.Tsujikawa et al</i>	$-0.007 < \xi < -0.0017$	<i>CMB</i>
<i>DGP Brane Inflation</i> [8]	<i>K.Nozari et al</i>	$\xi \geq 1.2 \times 10^{-2}$	<i>WMAP3</i>
<i>Our Model</i> ⁴	<i>K.Nozari et al</i>	$\xi \leq -0.1666, \xi \geq 0.01$	<i>WMAP3</i>
<i>Our Model</i> ⁵	<i>K.Nozari et al</i>	$0.271 \leq \xi \leq 0.791$	<i>WMAP3</i>

¹ Hubble Scale as Infra-Red Cutoff

² Solar System Explorer

³ Cosmological Radio Sources

⁴ For $V(\phi) = \lambda\phi^4$ in tree-level

⁵ For $V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{3m_{pl}^2}}\phi\right)$.

References

- [1] V. Faraoni, *Phys. Rev. D* **53** (1996) 6813
- [2] V. Faraoni, *Phys. Rev. D* **62** (2000) 023504
- [3] J. -P. Uzan, *Phys. Rev. D* **59** (1999) 123510
- [4] M. Bouhamdi-Lopez and D. Wands, *Phys. Rev. D* **71** (2005) 024010
- [5] A. A. Grib and W. A. Rodrigues, *Gravit. Cosmol.* **1** (1995) 273
- [6] E. Elizalde and S. D. Odintsov, *Phys. Lett. B* **333** (1994) 331
- [7] K. Nozari, *JCAP* **09** (2007) 003

- [8] K. Nozari and B. Fazlpour, arXiv:0708.1916
- [9] S. Koh, S. P. Kim and D. J. Song, *Phys. Rev. D* **72**, (2005) 043523
- [10] M. Ito, *Europhys. Lett.* **71** (5) (2005) 712
- [11] T. Chiba, *Phys. Rev. D* **60** (1999) 083508
- [12] Q. -G. Huang, M. Li and J. -H. She, *JCAP* **0611** (2006) 010,
- [13] M. Li, *JCAP* **10** (2006) 003
- [14] Y. Watanabe and E. Komatsu, *Phys. Rev. D* **75** (2007) 061301
- [15] N. Sakai and J. Yokoyama, *Phys. Lett. B* **456** (1999) 113
- [16] S. Mukaigawa, T. Muta and S. D. Odintsov, *Int. J. Mod. Phys. A* **13** (1998) 2739
- [17] D. N. Spergel *et al*, *ApJS* **170** (2007) 377
- [18] S. Tsujikawa, *Phys. Rev. D* **62** (2000) 043512
- [19] G. Esposito-Farese, D. Polarski, *Phys. Rev. D* **63** (2001) 063504
- [20] C. Schmid , J. P. Uzan and A. Riazuelo, *Phys. Rev. D* **71** (2005) 083512
- [21] N. Makino and M. Sasaki, *Prog. Theor. Phys.* **86** (1991) 103
- [22] R. Fakir, S. Habib and W. G. Unruh, *Astrophys. J.* **394** (1992) 396
- [23] E. Komatsu and T. Futamase, *Phys. Rev. D* **59** (1999) 064029
- [24] A. Liddle and D. Lyth, *Cosmological Inflation and Large-Scale Structure*, Cambridge University Press, 2000
- [25] D. I. Kaiser, *Phys. Rev. D* **32**(1995) 4295
- [26] A. R. Liddle, *Phys. Lett. B* **340**, (1994) 23
- [27] E. D. Stewart and D. H. Lyth, *Phys. Lett. B* **302** (1993) 171
- [28] S. M. Carroll, *Phys. Rev. Lett.* **81** (1998) 3067
- [29] T. Futamase, T. Rothman, R. Matzner, *Phys. Rev. D* **39** (1989) 405
- [30] S. Hancock *et al*, *Nature (London)* **367** (1994) 333

- [31] A. Bilandzic, T. Prokopec, arXiv:0704.1905
- [32] S. Tsujikawa, B. Gumjudpai, *Phys. Rev. D* **69** (2004) 123523